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NEW ISSUES IN QUARK-LEPTON SYMMETRY[1]

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Abstract

Two models incorporating different forms of spontaneously broken quark-lepton symmetry are discussed. Both models are constructed so that quark-lepton symmetry can be broken at as low an energy scale as phenomenology allows, thus maximising their testability. The first model uses the Pati-Salam gauge group SU(4)⊗SU(2)$_L$⊗SU(2)$_R$. The main analysis presented here concerns large threshold corrections to the $m_b = m_\tau$ tree-level mass relation. The second model is a new and simplified theory using discrete quark-lepton symmetry which has a very economical Higgs sector and elegantly avoids a potential quark-lepton mass relation problem.

1 Introduction

The idea that quarks and leptons are related in a pairwise fashion has a long history. In the late 1950's Marshak discussed an evident "quark-hadron correspondence" whereby the leptons e, ν_e and μ were associated with the hadrons p, n and Λ. This turned into a "quark-lepton correspondence" between e, ν_e, μ and d, u, s once quark substructure was revealed. With the discovery of ν_μ, the quark-lepton correspondence idea was used to postulate the existence of a fourth quark c. This remarkable prediction bore fruit with the discovery of charm in the mid 1970's. The correspondence between quarks and leptons was subsequently found to also hold for third generation fermions.

The pairwise association between quarks and leptons is so striking that it makes sense to suppose that the fundamental Lagrangian of the world actually treats quarks and leptons on an equal footing, with their disparate properties ascribed to a spontaneous violation of the exact quark-lepton symmetry. Such a symmetry can either be continuous or discrete. Candidates for a continuous symmetry of this nature are Pati-Salam SU(4) [1] and the various grand unifi-

[1] Invited talk given at Valencia 95, to appear in the Proceedings.

cation groups. Alternatively, a discrete symmetry between quarks and leptons can be introduced by postulating a spontaneously broken colour SU(3) group for leptons [2].

In this talk I will describe two pieces of recent work in this area. The first piece of work analyses some of the implications of having Pati-Salam SU(4) broken at the lowest energy scale that phenomenology allows [3] (1000 TeV). I will discuss how best to treat the neutrino mass problem, followed by the core of the analysis which is a detailed computation of radiative corrections to the tree-level mass relation $m_b = m_\tau$. I will show here that threshold corrections due to charged Higgs boson graphs can be very large. The second piece of work is a reconstruction of the quark-lepton discrete symmetry idea in such a way that the Higgs sector of the model is kept as simple as possible. I will also demonstrate that the resulting theory deals with the quark-lepton mass relation "problem" in an elegant way.

2 Low-scale Pati-Salam SU(4)

The following discussion is based on Ref.[4]. Under the Pati-Salam gauge group $G_{\rm PS}$ =SU(4)$\otimes$SU(2)$_L$$\otimes$SU(2)$_R$, a fermion generation appears as follows:

$$f_L \sim (4,2,1), \qquad f_R \sim (4,1,2). \tag{1}$$

The SU(4) factor transforms quarks into leptons in a continuous way. It contains the subgroup SU(3)$_c$$\otimes$U(1)$_{B-L}$. The linear combination $2I_R + (B-L)$, where I_R is the diagonal generator of SU(2)$_R$, is identified as weak hypercharge Y. I do not impose a discrete left-right symmetry on the model.

The model has a two stage symmetry breaking chain. At a relatively high scale $M_{\rm PS}$ SU(4) and SU(2)$_R$ are broken simultaneously, which is followed by electroweak symmetry breaking at its usual scale. Let us discuss electroweak symmetry breaking first. This is achieved by introducing a complex bidoublet Higgs field $\Phi \sim (1,2,2)$. The Yukawa Lagrangian

$$\mathcal{L}_{\rm Yuk} = \lambda_1 {\rm Tr}[\overline{f}_L \Phi f_R] + \lambda_2 {\rm Tr}[\overline{f}_L \Phi^c f_R] + {\rm H.c.} \tag{2}$$

where $\Phi^c \equiv \tau_2 \Phi^* \tau_2$ yields the tree-level mass relations

$$m_b = m_\tau \quad \text{and} \quad m_t = m_{\nu_3}^{\rm Dirac} \tag{3}$$

given a nonzero vacuum expectation value for Φ (the fermion multiplets are written as 2×4 matrices in the above). Similar relations hold for the first two generations, but the present analysis will confine itself to the third generation. The deep problem of generation structure will not be tackled here.

What is one to do with these mass relations? The equality between m_t and the neutrino Dirac mass is usually taken care of through the see-saw mechanism [5] by using a $\Delta \sim (10,1,3)$ Higgs multiplet to generate a large Majorana mass

for the right-handed tau neutrino. This also breaks SU(4)$\otimes$SU(2)$_R$ down to SU(3)$_c\otimes$U(1)$_Y$. However, the cosmological closure bound on the light mass eigenstate $m_t^2/\langle\Delta\rangle < 30$ eV leads to $\langle\Delta\rangle \sim 10^{12}$ TeV. This is much higher than the phenomenological lower bound of 1000 TeV, and thus use of the standard see-saw mechanism would undermine the motivation for the analysis.

An interesting alternative is to use what I will call the "3×3 see-saw mechanism" [6]. One introduces a singlet fermion $S_L \sim (1,1,1)$ and the simple Higgs multiplet $\chi \sim (4,1,2)$. The non-electroweak Yukawa Lagrangian

$$\mathcal{L}_{\text{Yuk}} = n\overline{S}_L \text{Tr}[\chi^\dagger f_R] + \text{H.c.} \tag{4}$$

when combined with the electroweak Yukawa terms above yield a neutrino mass matrix of the form

$$\begin{pmatrix} 0 & m_t & 0 \\ m_t & 0 & n\langle\chi\rangle \\ 0 & n\langle\chi\rangle & 0 \end{pmatrix} \tag{5}$$

in the $[\nu_L, (\nu_R)^c, S_L]$ basis. This produces one massless eigenstate which we identify with the standard neutrino, and a massive Dirac neutrino. For $n\langle\chi\rangle \gg m_t$, the massless state has approximately standard electroweak interactions. Because the light eigenstate is massless for all values of the nonzero entries of the mass matrix, $\langle\chi\rangle$ can be reduced to about 1000 TeV.

The $m_b = m_\tau$ relation receives radiative corrections of two types: those associated with large logarithms which are best calculated through renormalisation group equations, and threshold corrections which depend on either heavy/heavy or light/light mass ratios. It is interesting that renormalisation group evolution using standard particles in the loops merges m_b with m_τ at about 1000 TeV [7]! Therefore if Pati-Salam SU(4) is important for the fermion mass problem, an indirect indication through $K^0 \to e^\pm\mu^\mp$ may be just around the corner. But this renormalisation group result is only relevant if threshold corrections are not too large. It turns out that there is a generically large high-scale threshold correction due to loops containing the physical charged Higgs boson. See Ref.[4] for details. These graphs produce a threshold correction given by

$$m_\tau - m_b \simeq -\frac{1}{16\pi^2}\frac{m_s^2 - m_t^2}{m_H^2 - m_s^2}\frac{m_t(m_t - m\sin 2\omega)(m_t \sin 2\omega - m)}{(u_1^2 + u_2^2)\cos^2 2\omega}\ln\frac{m_s^2}{m_H^2}, \tag{6}$$

where m_s is the heavy Dirac neutrino mass, m_H is the charged Higgs boson mass, m is the "common" tree-level mass for b and τ, $u_{1,2}$ are the electroweak breaking VEVs and $\tan\omega = u_2/u_1$.

This threshold correction can clearly produce a mass difference between m_τ and m_b of the order of a GeV, provided an accidental cancellation between m and $m_t \sin 2\omega$ does not occur. The "common" mass m of τ and b at M_{PS} must be about the same as the measured m_τ, namely 1.8 GeV. The above threshold correction can therefore alter the initial ratio m_b/m_τ by up to 50%.

This correction is thus as numerically significant as those incorporated through the renormalisation group. The sign of the correction depends on the unknown parameter ω and therefore cannot be predicted. It can either raise or lower the mass ratio by up to 50%.

This calculation demonstrates that generally speaking one must take care in the use of renormalisation group evolution to predict low-energy masses.

3 New and improved quark-lepton symmetric model

The idea of discrete quark-lepton symmetry was introduced in Ref.[2]. It is based on the gauge group

$$G_{q\ell} = \mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X, \tag{7}$$

where $\mathrm{SU}(3)_\ell$ is leptonic colour, $\mathrm{SU}(3)_q$ is ordinary colour and X is an Abelian charge different from Y. A generation of fermions is placed into the following multiplet pattern:

$$\begin{array}{lll} Q_L \sim (1,3,2)(1/3), & u_R \sim (1,3,1)(4/3), & d_R \sim (1,3,1)(-2/3), \\ F_L \sim (3,1,2)(-1/3), & E_R \sim (3,1,1)(-4/3), & N_R \sim (3,1,1)(2/3). \end{array} \tag{8}$$

The standard leptons lie within F_L, E_R and N_R in a manner specified by the way $G_{q\ell}$ is spontaneously broken. The multiplet assignment above allows us to define a discrete quark-lepton symmetry by

$$\begin{array}{ccc} Q_L \leftrightarrow F_L, & u_R \leftrightarrow E_R, & d_R \leftrightarrow N_R, \\ G_q^\mu \leftrightarrow G_\ell^\mu & \text{and} & C^\mu \leftrightarrow -C^\mu, \end{array} \tag{9}$$

where $G^\mu_{q,\ell}$ are the gauge bosons of $\mathrm{SU}(3)_{q,\ell}$ and C^μ is the gauge boson of $\mathrm{U}(1)_X$.

The original quark-lepton symmetric models were based on $G_{q\ell}$ breaking achieved through the Higgs multiplet $\chi \sim (3,1,1)(2/3)$ that appears in the Yukawa Lagrangian

$$\mathcal{L}_{\text{Yuk}} = h_1 \overline{(F_L)^c} F_L \chi + h_2 \overline{(E_R)^c} N_R \chi + \text{H.c.} \tag{10}$$

(A quark-lepton symmetric partner for χ is also introduced with a corresponding Yukawa Lagrangian.) A nonzero VEV for χ breaks $G_{q\ell}$ down to $\mathrm{SU}(2)' \otimes G_{SM}$, where $\mathrm{SU}(2)'$ is a subgroup of leptonic colour that is necessarily unbroken. The weak hypercharge is identified as $Y = X + T_8/3$, where $T_8 = \mathrm{diag}(-2,1,1)$ is a generator of leptonic colour. The standard leptons are then the $T_8 = -2$ components, while the $T_8 = 1$ states are exotic fermions.

The minimal electroweak Higgs sector consists of a single doublet $\phi \sim (1,1,2)(1)$ as in the Standard Model. Under discrete quark-lepton symmetry it

transforms into its charge conjugate field $\phi^c = i\tau_2\phi^*$. The electroweak Yukawa Lagrangian

$$\mathcal{L}_{\text{Yuk}} = \lambda_1(\bar{Q}_L u_R \phi^c + \bar{F}_L E_R \phi) + \lambda_2(\bar{Q}_L d_R \phi + \bar{F}_L N_R \phi^c) + \text{H.c.} \tag{11}$$

then yields the tree-level mass relations $m_u = m_e$ and $m_d = m_\nu^{\text{Dirac}}$.

The second of these relations can be made phenomenologically acceptable by using the standard see-saw mechanism through the analogue of the Higgs field Δ of the Pati-Salam model discussed previously. In this case, $\Delta \sim (\bar{6}, 1, 1)(-4/3)$. The $m_u = m_e$ relation can only be dealt with by adding a second electroweak Higgs doublet which allows one to completely evade mass relations between quarks and leptons.

A Higgs sector consisting of two ϕ's, a χ and a Δ is rather cumbersome. It turns out that there is a much simpler way of constructing the model, which I now describe. This work is more fully explained in Ref.[8] which in turn is based in part on Ref.[9]. The new Higgs sector consists of only one ϕ and two χ-like fields I will call χ and ξ (plus their discrete symmetry partners). There is no Δ field because I am again going to use singlet fermions S_L together with an analogue of the 3×3 see-saw mechanism employed in the previous section. The VEV pattern for χ and ξ is the most general one possible, namely $\langle\chi\rangle = (v, 0, 0)^T$ and $\langle\xi\rangle = (w_1, w_2, 0)^T$ which completely breaks leptonic colour SU$(3)_\ell$. There is no unbroken SU$(2)'$ subgroup, and all components of the leptonic triplets become integrally charged [$Y = X + T_8/3 - T_3$ here where $T_3 = \text{diag}(0, 1, -1)$]. The standard leptons are then a linear combination of more than one leptonic colour.

The Yukawa Lagrangian of the model contains the electroweak Yukawa terms of Eq. 11, the non-electroweak terms of Eq. 10 extended to include ξ in an obvious way, together with mixing terms between the S_L's and N_R's given by

$$\mathcal{L}_{\text{Yuk}} = n_1 \overline{S}_L \chi^\dagger N_R + n_2 \overline{S}_L \xi^\dagger N_R + \text{H.c.} \tag{12}$$

The neutrino mass matrix consists of a 7×7 version of the 3×3 see-saw mechanism used in my Pati-Salam model. I will not display it here, except to say that the electroweak breaking entries in it are specified in terms of the quark masses and that the lightest mass eigenstate has close to standard electroweak interactions.

The charged lepton mass matrix is given by

$$\begin{pmatrix} m_u & 0 & M_3 \\ 0 & m_u & M_1 \\ M_4 & M_2 & m_d \end{pmatrix} \tag{13}$$

in the basis (e_1, e_2, e_3) where the subscript labels the three leptonic colours. The M_i are large masses proportional the the VEVs of χ and ξ. The smallest mass eigenvalue is given by $m_e = m_u \cos(\beta_1 - \beta_2) \leq m_u$ where $\tan\beta_1 \equiv M_2/M_4$ and

$\tan\beta_2 \equiv M_1/M_3$. Note that the $m_u = m_e$ mass relation does not arise here, and furthermore that m_e is necessarily less than m_u assuming large M_i and sufficiently small intergenerational mixing (this type of result was first found in Ref.[9]). It is straightforward to check that the fields corresponding to m_e have the correct electroweak interactions to a good approximation.

This reconstruction of the idea of discrete quark-lepton symmetry produces a simpler model with regard to the Higgs sector than previous versions, and it has no quark-lepton mass relation problems. In this sense the above theory can be regarded as an improvement on earlier models.

Acknowledgments

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